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DIA- AND PARAMAGNETOPHORESIS OF MICROPARTICLES NEAR A SHORT MAGNETIZED CYLINDER

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The motion of dia- and paramagnetic microparticles near a short magnetized cylinder and the conditions of registration and of spatial separation of diamagnetic particles by the value of magnetic susceptibility have been studied.

Keywords: *magnetic separation, magnetophoresis, numerical simulation.*

Introduction. The sorting of objects by their magnetic properties is widely used in various areas of the economy — from the mining-concentrating industry and purification of fluids to cell biology. The first works on the magnetic separation of cells are related to the 70–80s of the last century [1–5]. The method of immunomagnetic separation based on the binding of certain types of cells with small magnetic particles on the surface of which there are highly specific antibodies has undergone the greatest development and found practical application (see [6]). Direct magnetic assortment of cells and, on the whole, magnetophoretic processes in cell suspensions present a rather complex, as yet inadequately studied, problem. Investigations in this direction have become highly topical because of the recently observed intense development of medicinal cell technologies. The possibilities of magnetic methods in the separation of cells are evidenced by recent investigations in which substantial differences in the magnetic properties of one type of cells, such as erythrocytes [7] and insulin-producing cells of the pancreas of rubbit [8], have been established, as well as reliably recorded changes in the magnetic properties of cells of the spleen of mice as a result of the development of a malignant tumor in them (Ehrlich ascitis carcinoma) [9].

The motion of cells suspended in a liquid on exposure to an inhomogeneous external field originates if there is a difference in the magnetic susceptibilities of the cells and liquid. In their magnetic properties biological cells, just as physiological aqueous solutions, relate to diamagnetics. The absolute value of the magnetic susceptibility of cells is of the order of 10^{-6} (CGSM units), whereas the difference between the susceptibilities of cells and physicological solutions is even smaller. However, this difference can be markedly increased by introducing paramagnetic salts into a solution [10]. In any case, to create an appreciable magnetic force, external magnetic fields of high strength and strong fine-scale inhomogeneity are needed. In practice, such conditions are created near a small-size ferromagnetic body put into a strong homogeneous field. At the present time the theoretical foundation of ferromagnetic investigations is based on calculation of the motion of small weakly magnetic particles near a magnetized body of simplest geometry, namely, an infinite, over the length, transversely magnetized circular cylinder [11–16]. Traditionally, a problem on the conditions of capture, by an infinite cylinder, of the particles moving in a liquid along or across its axis is considered. The problem considered in the present work initially was motivated by the search for the optimal configuration of a magnetophoretic cell to determine the magnetic properties of cell ensembles. Earlier we considered a cell that represented a vertical plane slit with a long magnetic rod installed at its vertical end face; this rod was magnetized by a transverse homogeneous field in the direction of the slit plane. In this situation, the magnetic force acts in the slit plane across the gravitational force, whereas the magnetic susceptibility of a particle can be recovered with high accuracy by the results of recording its extended plane trajectory [17]. As a modification of this approach, we considered a plane motion of particles precipitating under gravity near a short vertical circular cylinder magnetized in the plane of motion. It turned out that the behavior of particles under these conditions has characteristic features that are of definite interest and open up new prospects of direct magnetic assortment of multicomponent suspensions of diamagnetic particles. The results of investigation of magnetophoresis near a short magnetized cylinder are presented in this work.

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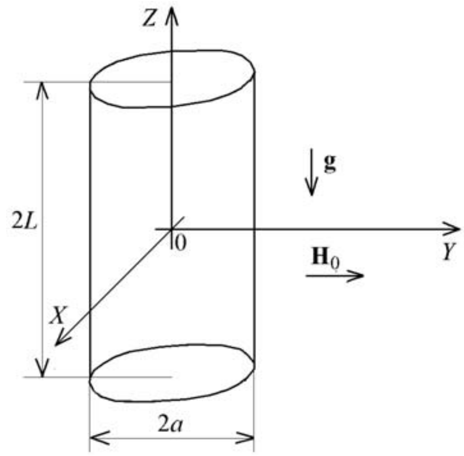


Fig. 1. Geometry of the problem.

Statement of the Problem and Calculation of Magnetic Field. The geometry of the problem is presented in Fig. 1. We consider the motion of a weakly magnetic particle in a viscous fluid caused by the action of the gravitational and high-gradient magnetic fields. To set up a high-gradient field, a vertical ferromagnetic cylinder of length $2L$ and diameter $2a$ is magnetized up to saturation by an external homogeneous magnetic field $\mathbf{H}_{ex}$ applied across the cylinder axis. We introduce a coordinate system with the origin at the geometric center of the cylinder (Fig. 1) and consider the motion of particles in the plane $X = 0$. The resultant field in the system can be represented as a sum of the magnetizing homogeneous field and the field $\mathbf{H}'$ produced by the magnetized cylinder.

Using the cylinder radius as the scale of the distance, we determine the dimensionless coordinates $x = X/a$, $y = Y/a$, and $z = Z/a$. Let B (x_B, y_B, z_B) be the point belonging to the cylinder. Then the infinitely small volume of the cylinder constructed near the point B with magnetization equal to the saturation magnetization of the ferromagnetic material I_s creates, at an arbitrary point A (x, y, z), the field strength defined by the relation

$$d\mathbf{H}'(\mathrm{B}, \mathrm{A}) = -I_s \mathbf{K}(\mathrm{B}, \mathrm{A})\, dx_B dy_B dz_B\,,$$

$$\mathbf{K}(\mathrm{B}, \mathrm{A}) = \frac{1}{r_{BA}^3}\left[\mathbf{e} - 3\frac{(\mathbf{e}\mathbf{r}_{BA})\,\mathbf{r}_{BA}}{r_{BA}^2}\right], \quad \mathbf{r}_{BA} = (x - x_B)\,\mathbf{i} + (y - y_B)\,\mathbf{j} + (z - z_B)\,\mathbf{k}\,, \quad \mathbf{e} = \frac{\mathbf{H}_{ex}}{|\mathbf{H}_{ex}|}\,. \tag{1}$$

The field $\mathbf{H}'$ will be determined by integrating Eq. (1) over the cylinder volume:

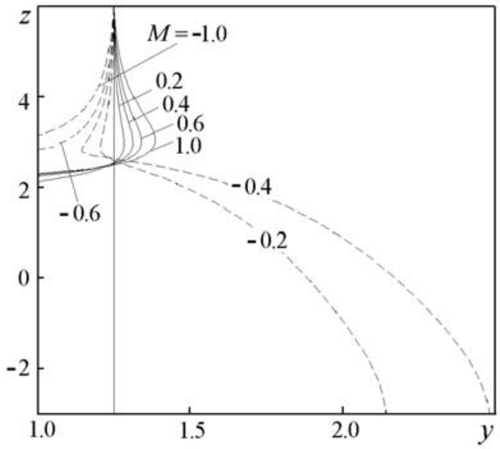
$$\mathbf{H}'(x, y, z) = -I_s \iiint_v \mathbf{K}\, dx_B dy_B dz_B\,. \tag{2}$$

The considered external field–cylinder system is symmetric about the plane $x = 0$. Consequently, in this plane the x-component of the field is equal to zero. Scaling the field strength by the quantity $2\pi I_s$ ($\mathbf{h} = \mathbf{H}/2\pi I_s$) for the nonzero component of the eigenfield in the plane $x = 0$, we have

$$h'_y = -\frac{1}{2\pi}\iiint_v \frac{1}{r_{BA}^3}\left[1 - 3\frac{(y - y_B)^2}{r_{BA}^2}\right] dx_B dy_B dz_B\,, \quad h'_z = \frac{1}{2\pi}\iiint_v \frac{3}{r_{BA}^5}(y - y_B)(z - z_B)\, dx_B dy_B dz_B\,,$$

$$r_{BA}^2 = x_B^2 + (y - y_B)^2 + (z - z_B)^2\,. \tag{3}$$

Having computed integrals (3) over the variables z_B and x_B in an explicit form, we represent these equations as follows:



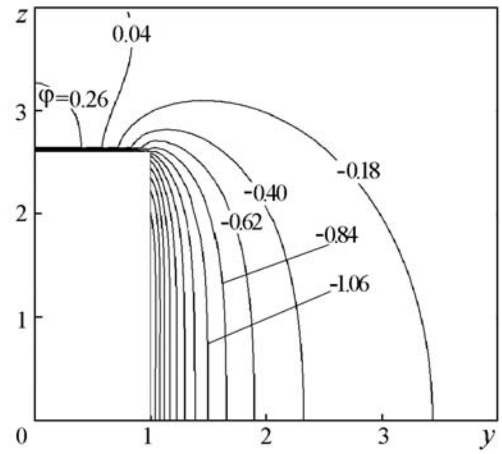


Fig. 2. Trajectories of particles for different values of the magnetophoretic parameter M (impact parameter $\delta = 1.25$; half-length of the cylinder $l = 2.6$ of the radius, dimensionless strength of the field $P = 1.5$).

Fig. 3. Equipotential lines of the magnetophoretic potential φ of a transversely magnetized cylinder in the plane $x = 0$ (half-length of the cylinder $l = 2.6$ of the radius, dimensionless strength of the field $P = 1.5$).

$$h'_y = \int_{-1}^{1} (f_2 (y_B, y, -z, l) + f_2 (y_B, y, z, l))\, dy_B\,, \quad h'_z = \int_{-1}^{1} (y - y_B)\,(f_1 (y_B, y, -z, l) - f_1 (y_B, y, z, l))\, dy_B\,. \tag{4}$$

Here $l = L/a$ is the dimensionless half-length of the cylinder:

$$f_1 (y_B, y, z, l) = \frac{1}{\pi}\left(\frac{1 - y_B^2}{1 - 2yy_B + y^2 + (l + z)^2}\right)^{1/2} \frac{1}{(y - y_B)^2 + (l + z)^2}\,;$$

$$f_2 (y_B, y, z, l) = \frac{(l + z)\,(1 - y_B^2 + 2\,(y - y_B)^2 + (l + z)^2)}{1 - 2yy_B + y^2} f_1 (y_B, y, z, l)\,.$$

The motion of a particle in the inertialess approximation is described by the equation

$$\mathbf{F}_m - 3\alpha\pi d\eta \frac{d\mathbf{R}}{dt} + \mathbf{g}\Delta\rho V = 0\,, \quad \Delta\rho = \rho - \rho_0\,, \tag{5}$$

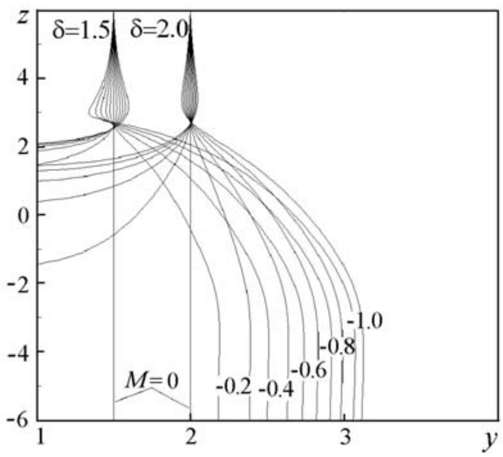
that expresses the condition of the mutual compensation of the magnetic, sedimentation, and viscous forces. Provided that the scale of the magnetic field inhomogeneity is great as compared to the dimensions of the particle, the magnetic force is given by the relation [18]

$$\mathbf{F}_m = \frac{1}{2}\Delta\chi V\nabla\mathbf{H}^2\,, \quad \Delta\chi = \chi - \chi_0\,. \tag{6}$$

Following [17], we will introduce the magnetophoretic potential of the field Φ according to the equation

$$\mathbf{F}_m = -\nabla\Phi\,, \quad \Phi = -\frac{1}{2}\Delta\chi V\mathbf{H}^2\,. \tag{7}$$

Having discarded the quantity ($\sim\mathbf{H}_{ex}^2$) in the potential, we write



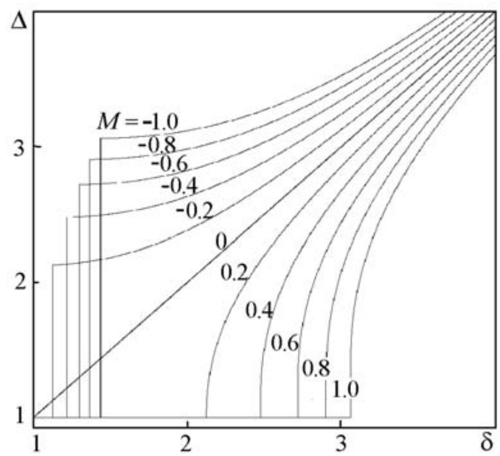


Fig. 4. Trajectories of particles for different values of the magnetophoretic parameter M and impact parameters δ = 1.5 and 2.0 (half-length of the cylinder l = 2.6 of the radius, dimensionless strength of the field P = 1.5).

Fig. 5. Function of the distance of exit of the impact parameter at different values of the magnetophoretic parameter M.

$$\Phi = -\frac{1}{2}\Delta\chi V\,(\mathbf{H}'^2 + 2\mathbf{H}_{\text{ex}}\mathbf{H}') = \Phi^*\varphi\,, \quad \varphi = -h_y'^2 - h_z'^2 - Ph_y'^2\,, \quad \Phi^* = 2\Delta\chi V\,(\pi I_s)^2\,, \quad P = \frac{|\mathbf{H}_{\text{ex}}|}{\pi I_s}\,. \tag{8}$$

Now we pass to dimensionless coordinates and break down the equation of motion (5) in the plane x = 0 into its components:

$$3\alpha\pi da\eta\,\frac{dz}{dt} = -\,\Delta\rho gV - \frac{2\Delta\chi V\,(\pi I_s)^2}{a}\frac{\partial\varphi}{\partial z}\,, \quad 3\alpha\pi da\eta\,\frac{dy}{dt} = -\frac{2\Delta\chi V\,(\pi I_s)^2}{a}\frac{\partial\varphi}{\partial y}\,. \tag{9}$$

Excluding the time in Eq. (9), we arrive at the following equation of the trajectory of the particle in the plane x = 0:

$$\frac{dz}{dy} = \left(\frac{\partial\varphi}{\partial y}\right)^{-1}\left[\frac{1}{M} + \frac{\partial\varphi}{\partial z}\right], \quad M = \frac{2\Delta\chi\,(\pi I_s)^2}{a\Delta\rho g}\,. \tag{10}$$

Analysis of the Motion of Particles. Let the particles precipitate under the action of the gravity force ($\Delta\rho > 0$) along the line offset from the cylinder axis by the distance $\delta > 1$ (impact parameter). The character of the influence of the cylinder is illustrated by Fig. 2, which presents the trajectories calculated at different values of the magnetophoretic parameter M in the range from −1 to 1 for the impact parameter δ = 1.25, dimensionless half-length of the cylinder l = 2.6, and for the value P = 1.5. The behavior of the particles is characterized by the following characteristic features. When approaching the cylinder, the paramagnetic particles ($M > 0$) are first repulsed by the cylinder, but then, as they approach the plane of the cylinder cut, they are attracted to it. The behavior of diamagnetic particles is opposite. Here, beginning from a certain value of $M < 0$, the capture of particles by the cylinder is observed. The diamagnetic particles that managed to avoid the capturing are repulsed by the cylinder and leave the zone of its influence at a certain distance Δ from the cylinder axis.

The above-described behavior of particles is connected with the character of distribution of the magnetophoretic potential whose distribution for the above-considered example is presented in Fig. 3 by the equipotential lines. The paramagnetic particles experience the action of the magnetic force directed to the side of the crowding of the equipotential lines, and the diamagnetic ones — to the side of their spreading out.

For magnetic separation, of interest is the specific feature of the influence of the cylinder on the motion of diamagnetic particles as shown in Fig. 4, which represents two families of the trajectories obtained for identical sets of values of the magnetophoretic parameter and different values of the impact parameter (δ = 1.5 and 2.0). This specific feature consists in the fact that the particles with identical values of the magnetophoretic parameter get closer, when they pass near the cylinder, with the difference between the coordinates of the escape decreasing with increase in the absolute value of M. The effect of the approach of diamagnetic particles with identical values of M at the exit can be described as

$$\Delta = f(\delta, M) . \tag{11}$$

The family of the curves $\Delta(\delta)$ obtained for a number of values of M is presented in Fig. 5. Corresponding to the horizontal straight line $\Delta = 1$ in this figure is the precipitation of particles on the cylinder surface. The sharp jump of the function $\Delta = f(\delta)$ for diamagnetic particles at small impact parameters corresponds to the above-described effect of the capture of them by the cylinder.

It is seen from Fig. 5 that with increase in the absolute value of the quantity $M < 0$ and decrease in the impact parameter the distance of the escape of particles depends more and more weakly on the impact parameter. Thus, diamagnetic particles with different values of M, which are incident on the cylinder as a wide stream, are separated at the exit depending on the values of M.

Conclusions. The results of investigation of the laws governing the motion of weak magnetic microparticles in the vicinity of a short magnetized cylinder point to the possibility of using the given geometry for spatial separation of diamagnetic particles and for the analysis of the distribution of them by magnetic properties.

NOTATION

a, radius of the cylinder, cm; d, diameter of a particle, cm; $\mathbf{e}$, unit vector in the direction of the external homogeneous field; $\mathbf{F}_m$, magnetic force, dyn; $\mathbf{g}$, free fall acceleration vector, cm·sec^{-2}; $\mathbf{H}$, magnetic field strength vector, Oe; $\mathbf{H}_{ex}$, external, homogeneous magnetic field strength vector Oe; $\mathbf{H}'$, magnetic self-field strength vector of a magnetized cylinder, Oe; $\mathbf{h}$, dimensionless magnetic field strength vector; h'_y, h'_z, components of the dimensionless strength vector of self-field of a magnetic cylinder; I_s, saturation magnetization of a cylinder, Gs; $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, unit coordinate vectors; $\mathbf{K}$, vector two-point function; L, half-length of a cylinder, cm; M, magnetophoretic parameter, dimensionless quantity; P, dimensionless external field strength; $\mathbf{R}$, radius vector, cm; r^2_{BA}, dimensionless distance between points A and B; t, time, sec; V, volume of a particle, cm^3; v, region of integration, corresponds to a cylinder geometrically; X, Y, Z, Cartesian coordinates, cm; x, y, z, dimensionless Cartesian coordinates; α, hydrodynamic parameter of the shape of a particle; Δ, exit distance, dimensionless quantity; δ, impact parameter, dimensionless quantity; $\Delta\rho$, difference between the densities of the particle and liquid, g·cm^{-3}; $\Delta\chi$, difference between the magnetic susceptibilities of the particle and liquid, dimensionless quantity; η, viscosity of a liquid, P; ρ, density of a particle, g·cm^{-3}; ρ_0, density of a liquid, g·cm^{-3}; Φ, magnetophoretic potential, erg; Φ^*, scale of magnetophoretic potential, erg; φ, dimensionless magnetophoretic potential; χ, magnetic susceptibility of a particle, dimensionless quantity; χ_0, magnetic susceptibility of a liquid, dimensionless quantity. Subscripts: 0, refers to a liquid; A, refers to point A; B, refers to point B; ex, external; m, magnetophoretic; s, saturation; x, y, z, components of a quantity along the corresponding coordinate.

REFERENCES

1. D. Melville, F. Paul, and S. Roath, High gradient magnetic separation of red cells from whole blood, *IEEE Trans.*, *Magn.*, **Mag-11**, No. 6, 1701–1704 (1975).
2. C. S. Owen, High gradient magnetic separation of erythrocytes, *Biophys. J.*, **22**, 171–178 (1978).
3. F. Paul, S. Roath, and D. Melville, Differential blood cell separation using a high gradient magnetic field, *Br. J. Hem.*, **38**, 273–280 (1978).
4. F. Paul, S. Roath, D. Melville, D. C. Warhurst, and J. O. S. Osisanya, Separation of malaria-infected erythrocytes from whole blood: use of a selective high gradient magnetic separation technique, *Lancet*, **2**, 70–71 (1981).
5. Yu. A. Plyavin' and É. M. Blum, Magnetic properties and para- and diamagnetophoresis of blood cells in high-gradient magnetic separation, *Magn. Gidrodinam.*, No. 4, 3–14 (1983).

6. M. Zborowski and J. J. Chalmers, Magnetic cell separation, *Lab. Tech. Biochem. Mol. Biol.*, **32**, 249–260 (2008).
7. M. Zborowski, G. R. Ostera, L. R. Moore, S. Milliron, J. J. Chalmers, and A. N. Schechter, Red blood cell magnetophoresis, *Biophys. J.*, **84**, 2638–2645 (2003).
8. B. É. Kashevskii, V. A. Goranov, A. M. Zholud', and A. V. Prokhorov, Magnetic sorting of β-cells, *Dokl. Nats. Akad. Nauk Belarusi*, **53**, No. 2, 69–71 (2009).
9. B. É. Kashevskii, T. I. Terpinskaya, A. M. Zholud', and V. A. Kul'chitskii, Information significance of magnetophoretic measurements to characterize cell suspensions, in: *Molecular and Cellular Principles of the Functioning of Biosystems*, *Int. Scient. Conf.*, Vol. 2, Minsk (2008), pp. 304–307.
10. C. H. Evans and W. P. Tew, Isolation of biological materials by use of erbium (III)-induced magnetic susceptibilities, *Science*, **213**, No. 4508, 653–654 (1981).
11. J. A. Oberteuffer, Magnetic separation: a review of principles devices, and applications, *IEEE Trans.*, *Magn.*, **Mag-10**, No. 2, 223–238 (1974).
12. J. Y. Hwang, M. Takayasu, F. J. Friedlaender, and G. Kullerud, Application of magnetic susceptibility gradients to magnetic separation, *J. Appl. Phys.*, **55**, No. 6, 2592–2594 (1984).
13. J. H. P. Watson, Magnetic filtration, *J. Appl. Phys.*, **44**, No. 9, 4209–4213 (1973).
14. R. Gerber, Theory of particle capture in axial filters for high gradient magnetic separation, *J. Phys. D: Appl. Phys.*, **11**, 2119–2129 (1978).
15. J. H. P. Watson, Theory of capture of particles in magnetic high-intensity filters, *IEEE Trans.*, *Magn.*, **Mag-11**, No. 5, 1597–1599 (1975).
16. W. F. Lawson, W. H. Simons, and R. P. Treat, The dynamics of a particle attracted by magnetized wire, *J. Appl. Phys.*, **48**, No. 8, 3213–3224 (1977).
17. B. É. Kashevskii, I. V. Prokhorov, S. B. Kashevskii, P. Yu. Istomin, and E. N. Aleksandrova, Magnetophoresis and magnetic susceptibility of HeLa tumor cells, *Biofizika*, **51**, No. 6, 1026–1032 (2006).
18. L. D. Landau and E. M. Lifshits, *Electrodynamics of Continuous Media* [in Russian], Nauka, Moscow (1982).